

Top and Higgs Mass from Cancellation Conditions

K. Just and L. S. The

Department of Physics, University of Arizona, Tucson AZ 85721

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We propose a way of connecting the masses of all those particles which couple to the “electroweak” Higgs scalar. Up to radiative corrections to be given later, we expect $m_t = 81$ GeV and $m_H = 66$ GeV. Our realistic (but flexible) cutoff is explained as far as needed.

1. Introduction

The success of the electroweak gauge theory [1] has led to a great revival of Quantum Field Theory. By this, however, many theorists do *not* mean a theory of quantized interacting fields [2]. One rather derives observable results from the “Green’s functions” specified by Feynman’s integration over classical fields [3]. All problems are thus reduced to the *choice* of the action, which then is evaluated in standard ways (numerically [4] or by perturbation [5]).

For us who still employ quantized interacting fields, a critical question is how these (and their products) can exist. Is it essential to construct [6] everything with *mathematical* rigor? Can supergravity remedy the divergencies, or will more recent ideas [7] be needed? Should we admit new *physical* structures such as a realistic cutoff [8]? Here let us explore the last possibility, since other avenues have been widely discussed.

The foundations of fields quantized in our sense shall include:

- (i) A *realistic cutoff* to make products of quantized fields exist even at coincident points.
- (ii) *Field equations* compatible with (i), derived from an action (of quantized interacting fields).

These features will be clarified below and in papers to follow. For instance, the cutoff will break conformal symmetry, but respect coordinate and gauge invariance. We also shall obey the axioms from the general theory of quantized fields [2] as far as possible.

In Sect. 2 we discuss the practical merits of the dimensional regularization, state our mass predictions and compare them with others. In Sect. 3 we partly explain our cutoff; in Sect. 4 we find a consequence for the standard Lagrangian. In Sect. 5 we derive cutoff integrals and cancellation conditions, in Sect. 6 we evaluate them and discuss the results. A comparison with perturbation theory, super-symmetry, and hierarchy problems is given in Section 7.

2. Mass Predictions

For most practical questions, the distinction between different foundations is so minor that often one is in doubt as to which of them are adopted by certain authors. Hence it is of special interest when two attitudes can be distinguished by merely *counting* their observable consequences. This happens in the problem of quadratic divergencies. A realistic cutoff at the mass Λ makes $i \int p^{-2} dp = (\pi\Lambda)^2$; but this integral *vanishes* under dimensional regularization. Hence the latter suppresses cancellation conditions provided by the cutoff.

Dimensional regularization remains an ingenious device for evaluating theories with prescribed couplings. This holds especially for physically incomplete models such as pure quantum electrodynamics, where cancellations of fermion and boson masses are impossible. In other words, working in $d \neq 4$ dimensions is *excellent* for circumventing issues inaccessible for too unrealistic models. We merely deny the fundamental significance one sometimes ascribes to dimensional regularization.

The effects of a gauge invariant cutoff on perturbation theory and on the metric in the Hilbert

Reprint requests to Dr. K. Just, Department of Physics, University of Arizona, Tucson AZ 85721, USA.

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space being discussed elsewhere [9], below we obtain an equation for the standard Higgs field. For this scalar and for the heaviest quark, that equation leads to

$$\begin{aligned} 40 \text{ GeV} < m_H < 90 \text{ GeV} \quad \text{and} \\ 70 \text{ GeV} < m_t < 90 \text{ GeV}. \end{aligned} \quad (1)$$

These estimates do not violate confirmed experimental or theoretical bounds.

From asymptotic freedom, Kubo, Siebold, and Zimmermann [10] have derived

$$m_H = 61 \text{ GeV} \quad \text{and} \quad m_t = 81 \text{ GeV}. \quad (2)$$

Our very different arguments for (1) are *compatible* with those for (2), and so are the results. Like us, the authors predicting (2) evaluated the well-confirmed Standard Model, although one generally regards it as far from fundamental. Since lower masses were expected and indicated previously [11], the agreement between (2) and (1) is remarkable.

3. The Assumed Cutoff

For clarity let us solely employ the Standard Model with minimal Higgs content [12]. Grand unifications and other extensions may be included by the reader. More work will be needed for further improvements we have in mind. They include the distinction between the values of coupling constants at low and high energies. The resulting corrections will be minor, and also the present omission of gravity can be motivated.

Much should be said about the relation of our proposal to other treatments of quadratic divergencies, to super-symmetry and hierarchy problems. Remarks are given in Section 7. For the calculations of Sects. 4 through 6, we need only the following comparison with a “too rigid” cutoff (such as that of Pauli-Villars with a precise mass Λ).

We consider the cutoff as a step towards a theory without sharp points in time and space. Though such a theory has often been envisioned [13], its direct formulation seems premature. We actually do not expect new mathematics (without a continuum); rather the relations between mathematical and physical “spaces” should be understood more realistically. In this sense, we regard any specific cutoff merely as a *representative* from a vast class of physically equivalent procedures.

This philosophy has the practical consequence that we require quadratic and logarithmic divergencies to cancel *separately*. From one equation for the Higgs field, we thus can deduce the vanishing of *two* expressions, (i) one which for $\Lambda \rightarrow \infty$ would diverge quadratically and (ii) another logarithmically divergent. Hence we attain (1) rather than a single relation between m_H and m_t . In other words, our cancellation conditions are stronger than those from dimensional regularization or from a rigid cutoff (though weaker than conditions which demand a supersymmetry).

4. A Basic Stationarity

That part of the Lagrangian which contains the “physical” Higgs field

$$\Phi^4 \equiv \Phi^0 - v \quad \text{with} \quad v = \langle \Phi^0 \rangle \approx 250 \text{ GeV}, \quad (3)$$

in the Standard Model without gravity can be written

$$\begin{aligned} \mathcal{L}_H = & \frac{1}{2} \Phi_\rho^H \Phi_\rho^H - v^{-1} \bar{\Psi} m \Psi \Phi^4 \\ & - \frac{1}{2} m_H^2 (\Phi^z \Phi^z / 2v + \Phi^4)^2. \end{aligned} \quad (4)$$

Using familiar notation such as

$$S_{\rho\rho} \equiv S_{00} - S_{11} - S_{22} - S_{33},$$

we denote any vacuum (expectation) value by $\langle \rangle$. We write $\Phi^H = \Phi^0$, Φ^r with $r = 1, 2, 3$, but $\Phi^z = \Phi^r$, Φ^4 for those hermitian Higgs components which obey $\langle \Phi^z \rangle = 0$.

In (4) we expressed the constants of the scalar self-interaction by the bare Higgs mass m_H . For this purpose, we invoked the tree approximation, leaving higher orders of the effective action [14] for later work. From the invariant combination $\Phi_\rho^H \Phi_\rho^H$ of the gauge covariant derivatives $\Phi_\rho^H = \Phi_{,\rho}^H + \dots$, we will only need

$$\begin{aligned} & \Phi_\rho^H \Phi_\rho^H \\ = & \dots + \frac{1}{4} \{ 2g^2 W_\rho^\dagger W_\rho + (g^2 + g'^2) Z_\rho Z_\rho \} \Phi^0 \Phi^0. \end{aligned} \quad (5)$$

From (4) with (5) we obtain Euler’s polynomial

$$\begin{aligned} E \equiv \delta \int \mathcal{L}_H / \delta \Phi^4 = & \dots - (m_H^2 / 2v) (\Phi^r \Phi^r + 3\Phi^4 \Phi^4) \\ & + \frac{1}{4} \{ 2g^2 W_\rho^\dagger W_\rho + (g^2 + g'^2) Z_\rho Z_\rho \} \cdot \Phi^0 \\ & - v^{-1} \bar{\Psi} m \Psi; \end{aligned} \quad (6)$$

the terms ignored here will not contribute below. After checking that the Faddeev-Popov fields c, \bar{c}

make our results gauge invariant, we have adopted the Landau gauge. Therefore (4) and (6) do not contain the c , \bar{c} , while the “unphysical” Higgs components Φ^1, Φ^2, Φ^3 remain massless.

By our realistic cutoff, Euler’s equations [15] and the equivalent stationarity of $\int \mathcal{L}$ become impossible (after infinite renormalizations, $\delta \int \mathcal{L} = 0$ does not hold either). The *vacuum* values of Euler’s polynomials vanish nevertheless, and also the well known stationarity of the effective action [14] implies $\delta \langle \int \mathcal{L} \rangle = 0$ (the simple proofs will follow elsewhere). Hence (6) must obey

$$v \langle E \rangle = \left(\frac{1}{2} v \right)^2 \langle 2 g^2 W_\phi^\dagger W_\phi + (g^2 + g'^2) Z_\phi Z_\phi \rangle - \left\langle \frac{1}{2} m_H^2 (\Phi^r \Phi^r + 3 \Phi^4 \Phi^4) + \bar{\Psi} m \Psi \right\rangle = 0. \quad (7)$$

Everything symbolized in (5) and (6) by dots, such as terms linear in (3), does not contribute in the present approximation.

5. Cancellation Conditions

In (7) we need the *complete* propagators of many basic, interacting fields. They occur because they directly couple to the Higgs field (3). Those unknown propagators can be approximated by the *bare* ones because they enter (7) mainly at high energies (this plausible feature of theories with a cutoff will be discussed later).

Therefore we obtain

$$\begin{aligned} (4\pi)^2 \langle \Phi^4 \Phi^4 \rangle &= i (A^2/\pi)^2 \int (p^2 - m_H^2)^{-1} (A^2 - p^2)^{-2} dp \\ &= A^2 - m_H^2 \ln (A/\mu)^2, \\ (4\pi)^2 \langle \Phi^r \Phi^r \rangle &= 3 A^2, \\ (4\pi)^2 \langle W_\phi^\dagger W_\phi \rangle &= 3 m_W^2 \ln (A/\mu)^2 - 3 A^2, \\ (4\pi)^2 \langle Z_\phi Z_\phi \rangle &= 3 m_Z^2 \ln (A/\mu)^2 - 3 A^2, \\ (4\pi)^2 \langle \bar{\Psi} m \Psi \rangle &= 4 \sum_f m_f^2 \{ m_f^2 \ln (A/\mu)^2 - A^2 \}, \end{aligned}$$

where f runs over all leptons and quarks. Having omitted terms finite for $A \rightarrow \infty$, we replaced the masses under the logarithms by an average μ . Thus (7) provides

$$(4\pi)^2 v \langle E \rangle = L \ln (A/\mu)^2 - Q A^2 = 0 \quad (8)$$

with

$$\begin{aligned} Q &= 3 m_H^2 + \frac{3}{4} (3 g^2 + g'^2) v^2 - 4 \sum_f m_f^2, \\ L &= \frac{3}{2} (m_H^4 + g^2 v^2 m_W^2) \\ &\quad + \frac{3}{4} (g^2 + g'^2) v^2 m_Z^2 - 4 \sum_f m_f^4. \end{aligned} \quad (9)$$

Since in non-integral dimensions d one gets $\int p^{-2} d^d p = 0$, dimensional regularization would yield (8) without $Q A^2$ (under the usual replacement of $(4-d)^{-1}$ by $\ln A/\mu$). Then the coupling constants would only be restricted by $L = 0$. On the other hand, an unambiguous cutoff at a *fixed* mass A would only demand that (8) as a whole equals zero. The ensuing condition $Q A^2 = L \ln (A/\mu)^2$ would in practice just require $Q = 0$, because

$$|L \ln (A/\mu)^2| A^{-2} \approx |L/A^2|$$

is much smaller than any term of Q .

In our opinion, however, the exact mass of the cutoff ought to be unimportant. Then (8) demands

$$L = 0 \quad \text{as well as} \quad Q = 0. \quad (10)$$

Hence the restriction $L = 0$ known from dimensional regularization must hold together with $Q = 0$, which in practice is required by any cutoff. In other words, our flexible cutoff imposes $Q = 0$ as an *additional* condition whereas a rigid cutoff would demand $Q = 0$ *instead of* $L = 0$.

6. Resulting Estimates

Within the accuracy expected here, only the *maximal* mass m_q of all the quarks contributes to $\sum m_f^2$ and $\sum m_f^4$. Since we are not sure whether the most massive quark is the top t or belongs to a fourth generation, we call it q . For an experimental search, it will be irrelevant whether q is the top quark or indicates another generation.

In any case, near our m_q *some* quark should be found. After the initial discovery of this most massive q , the less difficult observation of its branching ratios will reveal its nature. (In (1) we denoted the maximal quark mass by m_t , in order to defer these explanations until now, and because other arguments [16] favor 3 generations). Assuming 3 colors and expressing $g v$, $g' v$ as usual by m_W , m_Z , we now employ

$$\begin{aligned} g v &= 2 m_W, \quad (g' v)^2 = 4 (m_Z^2 - m_W^2), \\ \sum_f m_f^2 &= 3 m_q^2 \quad \text{and} \quad \sum_f m_f^4 = 3 m_q^4. \end{aligned}$$

When these are inserted, (9) gives

$$\begin{aligned} \frac{1}{3} Q &= m_H^2 + 2 m_W^2 + m_Z^2 - 4 m_q^2, \\ \frac{2}{3} L &= m_H^4 + 4 m_W^4 + 2 m_Z^4 - 8 m_q^4. \end{aligned} \quad (11)$$

Thus (10) and the observed [17] $m_W \approx 82$ GeV, $m_Z \approx 93$ GeV yield either

$$m_q \approx 124 \text{ GeV} \quad \text{and} \quad m_H \approx 199 \text{ GeV} \quad (12)$$

or

$$m_q \approx 81 \text{ GeV} \quad \text{and} \quad m_H \approx 66 \text{ GeV}. \quad (13)$$

Discarding (12) as too unlikely and anticipating radiative corrections, let us predict (1). Alternatively, we might neglect m_H and then evaluate $Q = 0$ alone (to invoke less philosophy). Then (11) would give $m_q \approx 74$ GeV, which lies safely within the estimate (1). For the Higgs mass, we have there assumed a wider tolerance because we feel less secure about the m_H from (13).

7. Hierarchy Problems

The relation here written $Q\Lambda^2 = 0$, with Q as in (9) and (11), has been proposed in perturbation theory [18] for avoiding an excessive “dynamical mass” for the Higgs scalar. The required *partial* cancellation of huge terms has rightly been criticized as unnatural. When a scalar couples to fields needed in Grand Unifications, such difficulties occur even under dimensional regularization (where the quadratically divergent integral $i \int dp (p^2 - M^2)^{-1}$ gets a value in the order of M^2). One may call any of these or other questions about the coexistence of very high and low masses a “hierarchy” problem.

Since $Q = 0$ demands a cancellation of (squared) boson and fermion masses, it has been said to suggest a super-symmetry [19]. This holds in particular when such cancellations are imposed for all values of the running coupling constants. In the same direction points our contention that (8) imposes not only $Q = 0$ but also $L = 0$. However, we maintain that the cutoff, although not determined with unreasonable precision, occurs not excessively far from Planck’s mass. In any case, the need for a super-symmetry does not arise here; we just claim that remarkable connections between bosons and fermions occur *without* this unobserved symmetry.

Also for us the “dynamical masses” of perturbation theory are suppressed, but solely as a *consequence* of $Q = L = 0$. Since these mass conditions came from the direct implication (7) of the fundamental stationarity $\delta \int \langle \mathcal{L} \rangle = 0$, those partial cancellations are no longer objectionable. Not all hierarchy problems are thus solved, because we do not yet derive small masses from large ones. Yet we reasonably connect the masses of the generally expected particles to those already observed, without invoking any particle not needed in the Standard Model.

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- [1] T. P. Cheng and L. F. Li, Gauge Theory of Elementary Particle Physics, Univ. Press, Oxford 1984.
- [2] R. Jost, The General Theory of Quantized Fields, Amer. Math. Soc., Providence 1965; R. F. Streater and A. S. Wightman, PCT, Spin & Statistics, and All That, Benjamin, New York 1964, Reading 1978.
- [3] L. H. Ryder, Quantum Field Theory, Univ. Press, Cambridge 1985, p. 202; — G. Velo and A. S. Wightman (Eds.), Renormalization Theory, Reidel, Dordrecht 1976; — Cheng, Li [1], p. 248.
- [4] M. Creutz, Quarks, Gluons, and Lattices, Univ. Press, Cambridge 1983; — Cheng, Li [1], p. 324.
- [5] L. Baulieu, Phys. Rep. **129**, 1 (1985); Cheng, Li [1], p. 56; L. H. Ryder [3], p. 164.
- [6] G. Velo and A. S. Wightman (Eds.), Constructive Quantum Field Theory, Springer, Berlin 1973; J. Glimm and A. Jaffe, Quantum Physics, A Functional Integral Point of View, Springer, New York 1981.
- [7] B. de Witt, Super-Manifolds, Univ. Press, Cambridge 1984; — Y. Nambu, Phys. Rep. **104**, 237 (1984); — M. J. Duff, B. E. W. Nilsson and C. N. Pope, Phys. Rep. **130**, 1 (1986).
- [8] J. Polchinski, Nucl. Phys. **B 231**, 269 (1984).
- [9] K. Just and P. Narra-more, Z. Naturforsch. **42a**, 236 (1987); — R. P. Feynman, Negative Probability, preprint, Pasadena 1985.
- [10] J. Kubo, K. Sibold, and W. Zimmermann, Nucl. Phys. **B 259**, 331 (1985).
- [11] G. Arnison et al., Phys. Lett. **147 B**, 493 (1984).
- [12] L. B. Okun, Leptons and Quarks, North Holland, Amsterdam 1983, pp. 185–235; — Cheng, Li [1], p. 336; L. H. Ryder [3], p. 307; J. Ellis [18], p. 162.
- [13] T. Padmanubjhan, Ann. Phys. **165**, 38 (1985). — G. Ludwig, Imprecision in Physics, in “Structure and Approximation in Physical Theories” (A. Hart-kämper and H. J. Schmidt, Eds.), Plenum, New York 1981, pp. 7–19; — D. Finkelstein, Space-Time Structure in High Energy Interactions, in “Fundamental Interactions at High Energies” (T. Gudehus, G. Kasier, and A. Perlmutter, Eds.), Gordon & Breach, New York 1969, pp. 324–343.

- [14] L. F. Abbott, *Acta Phys. Polon.* **B 13**, 33 (1982); — Cheng, Li [1], p. 189; L. H. Ryder [3], p. 387.
- [15] K. Just and L. S. The, *Found. Phys.* **16**, 1127 (1986); — T. Kugo and I. Ojima, *Suppl. Prog. Theor. Phys.* **66** (1979); — R. A. Brandt, *Fortschr. Physik* **18**, 246 (1970). — Velo, Wightman [3].
- [16] K. Just and C. Druce, *Pointlike Particles with Internal Structures*, preprint, Tucson 1986.
- [17] Particle Data Group, *Phys. Lett.* **170 B**, 1 (1986).
- [18] J. Ellis, *Phenomenology of Unified Gauge Theories*, in “Gauge Theories in High Energy Physics” (M. K. Gaillard and R. Stora, Eds.), North Holland, Amsterdam 1983, pp. 157–363; — M. Veltman, *Acta Phys. Polon.* **B 12**, 437 (1981).
- [19] M. Veltman [18], pp. 446–452; J. Ellis [18], pp. 317–326.